

# Karush-Kuhn-Tucker Proximity Measure for Convergence of Real-parameter Single and Multi-Objective Optimization

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The proximity of an obtained solution to theoretical optimum is always the first priority to any optimization algorithm developer. In constraint optimization problems, Karush-Kuhn-Tucker (KKT) optimality conditions can at most determine a binary decision about a solution - a likely candidate of being the optimum, or not an optimum. However, further information about a solution's proximity to the theoretical optimum either in the variable space or in the objective space would be valuable to the algorithm developer. Such information will also be valuable to the applicationists in assessing the solution's quality and acceptability as a final solution. In this talk, we shall present a KKT proximity measure (KKTPM) based on approximate KKT point concepts suggested in the literature. KKTPM exhibits a high correlation of the difference from optimal objective value to the KKTPM value., thereby making KKTPM a measure of convergence to the true optimum. The concept of KKTPM is also extended for multi-objective optimization to determine the proximity of a solution's objective vector from the theoretical efficient set, without any knowledge of the theoretical efficient set. Results on standard two to 15-objective test problems and on a few engineering design optimization problems will be shown. The computational burden of optimization-based KKTPM calculation is reduced using a direct approach in which an approximate KKTPM is computed. KKTPM has also been used to improve the performance of an evolutionary multi-objective optimization (EMO) algorithm in a unique way. Some other extensions and applications of the KKTPM will also be discussed to highlight the usefulness of the proposed KKTPM in single and multi-objective optimization. For further information, refer the following articles:

## References

- [1] Dutta, J., Deb, K., Tulshyan, R. and Arora, R. (2013). Approximate KKT points and a proximity measure for termination. *Journal of Global Optimization*, 56(4), 1463-1499.
- [2] Deb, K. and Abouhawwash, M. (2016). An Optimality Theory Based Proximity Measure for Set Based Multi-Objective Optimization. *IEEE Transactions on Evolutionary Computation*, 20(4), 515-528.